Report on AIM Workshop: Towards Relative Symplectic Field Theory

Jim Stasheff

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Abstract

September 24 to September 28, 2007: AIM workshop Towards Relative Symplectic Field Theory at the CUNY Graduate Center, New York City organized by Kai Cieliebak, Tobias Ekholm, Yakov Eliashberg, Kenji Fukaya, Dennis Sullivan, and Michael Sullivan.

See

http://www.aimath.org/ARCC/workshops/relsymplectic.html

for more details about the scope of the workshop.

September 24 to September 28, 2007 there took place an AIM workshop Towards Relative Symplectic Field Theory at the CUNY Graduate Center, New York City organized by Kai Cieliebak, Tobias Ekholm, Yakov Eliashberg, Kenji Fukaya, Dennis Sullivan, and Michael Sullivan

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I was able to attend September 25 and 25 so this is only a partial report. It was truly a workshop. There were lectures in the morning, but ones designed to provoke for the afternoon discussion and interaction between a variety of groups represented,roughly divided into algebra (in fact infinity algebra) and geometry (specifically symplectic geometry and in low dimensions).

Note: SFT usually meant Symplectic Field Theory but occasionally it meant String Field Theory. Some of the pictures are remarkably similar.

Here a rough sketch of what I did get to hear: A sub-theme was $D^2 = 0$.

Fukaya had already spoken but I did hear two of his 3 Os and the third (Oh) was also present.

Ohta: Whitehead's Theorem for A_∞-algebra

meaning when a weak homotopy equivalence implies a strong homotopy equivalence. Here extended/weak/curved A_∞-algebras were included and the ground field was extended to e.g. the Novikov ring. The proofs were by classical obstruction theory, so A_n-algebras for finite n played a role. A closed or even algorithmic formula was missing e.g. no analog of the 'tensor trick'.

Ono: Canonical models of filtered A_∞-algebras (including extended/weak/curved ones)

This involved a souped up version of Kadeishvili's original result (and alternative proofs later) that the homology of an A_∞-algebra inherits an A_∞-structure. For those who speak the language, call a homotopy for a deformation retract a 'Green's operator;! Here the proofs (following Kontsevich and Soibelman?) were by induction expressed in terms of rooted planar trees with stubs and an energy function.

Discussion:

Michael Sullivan: $D^2 = 0$ for algebras over an operad with two binary operations, $\circ$ and $\star$, and one unary inspired by pictures from symplectic geometry. Each operation to satisfy Jacobi and !! also their sum.

Sasha Voronov: Such a combination of operations, though with different pictues and different relations, occurs in dgBV algebras. cf. Zwiebach's String Field Theories.

9/26/07

Janko Latschev (work with Kai Cieliebak) $D^2 = 0$ for algebra inspired by Relative SFT.

Consider a Legendrian submanifold of dimension n in a contact manifold of Y dimension 2n+1which has no closed Reeb orbits. Reeb chords are arcs in Y with ends on the Legendrian (perhaps thought of as a brane). For n=1 and a Legendrian knot in R^3, the Reeb chords could be vertical segments in R^3. Goal: Find a Master equation and a solution in terms of moduli spaces of circles with marked points. Idea: A closed loop given as a map of a circle with twice as many marked points, alternating paths in the Legendrian with Reeb chords.

Algebra of chains on space of such:

$\{,\}$ given by glueing of such loops along a common Reeb chord (if any) which is then erased.

$[,]$ given by cutting and reglueing of such loops at a common point in the Legendrian by the usual reinterpretation of X as ¿ ¡ transforming to the top V and the bottom $\wedge$. Each operation satisfies Jacobi and !! also their sum.

In addition to the usual differential ∂, there is another one, δ, from inserting a degenerate example of the loops being considered, the kind which spends only an instant in the Legendrian.

That $D^2 = 0$ for $D = \partial + \delta$ follows as one would expect. That D is a derivation of the total $<,> = \{,\} + [,]$ is more subtle, involving cancelation of mixed terms.

Open questions:

1. Variance under cobordism up to some kind of homotopy?
2. Special case: n=1, Legendrian knot (or link) in R^3.
3. a 'herded' version??
4. augmentations? linearizations? removal of m_0?

Octav Cornea: Master equation for Cluster algebra

This was not so easy to summarize, but intriguing features are: Related to Morse theory, especially a le Fukaya

Pictures in terms of planar rooted trees with disks at internal nodes where the disks carry geometry.

Homotopy theory is difficult, even rationally so settle for homology.

Discussion:

Stasheff: Old stuff perhaps knot known to all present:

Kadeishvilli's transfer of algebra structure up to A_∞

Massey products are representatives

Gugenheim, Huebschmann et al transfer by the tensor trick

Huebschmann: L_∞ analog - tensor trick doesn't symmetrize

Markl: higher homotopy insights

Lie-massey brackets - cf. Retakh

Sullivan: A_∞-structures and other $D^2 = 0$ in terms of a formal manifold and vector field

Relation to correlations functions, e.g. Gromov-Witten, Donaldson

John Terilla: Master Equations as non-commutative deformations

Versal solution ¡–¿ specific A_∞-structure and specific quism (= quasi-iso)

For $H = H(A, d)$ and a splitting $H \subset A$, work in $A[[H^{dual}]]$

BV algebras and dgBV

Quantum Master Equation by perturbing d to $d + \hbar\Delta$

For a solution Γ, perturb further to $d + \hbar\Delta + [\Gamma, -]$

Theorem; There exists a versal Γ iff...

Barannikov and Kontsevich: special case in the presence of a $\partial, \bar{\partial} Lemma$ (Compare result of Heubschmann and Stasheff)

If there does NOT exist such a Γ, the obstructions can be expressed in terms of another L_∞-structure on H

Stasheff: review of joint work with Kajiura on OCHA = open closed strong homotopy algebra - inspired by string field theory